Université 8 mai 1945 Guelma

2eme ST

EXAMEN
THEORIE DU SIGNAL

20/20

1h:30 min   08/06/202

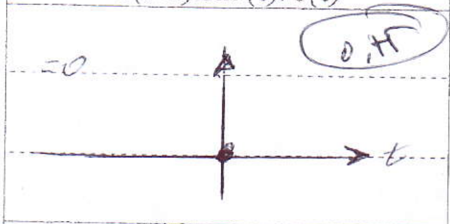
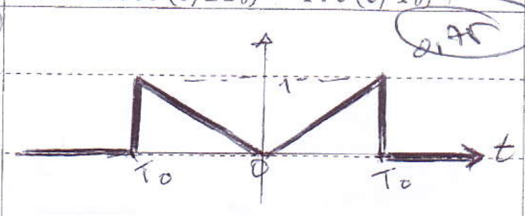
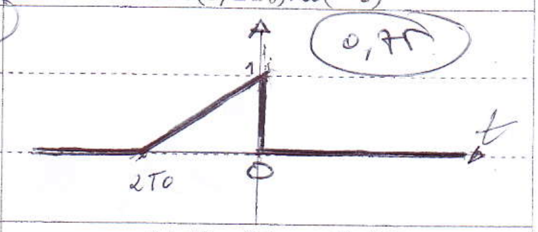
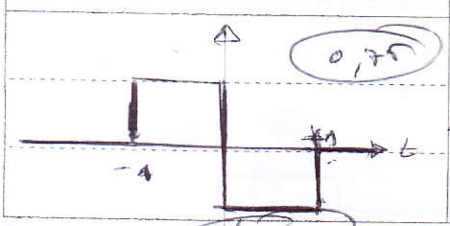
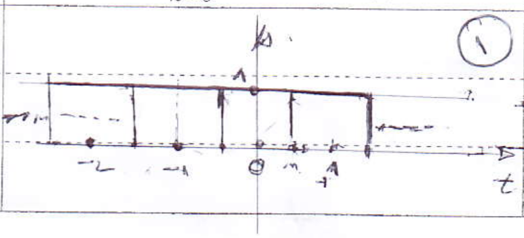
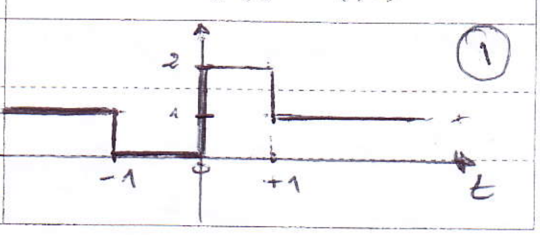
Nom……………………………………………………………………………

Code Anonyme ……….

Exercice1   05/05

Code Anonyme ……….

- *Représenter les signaux suivants en fonction du temps t*

$\delta(t-\pi).Sin(t).\delta(t)$	$Rect(t/2T_0) - Tri(t/T_0)$	$Tri(t/2T_0).u(-t)$
		
$d(tri(t))/dt$	$\sum_{k=0}^{\infty} Rect(t-k)$	$1+ Sign(t).rect(t/2)$
		

Exercice2   05/05

- *Calculer l'énergie et la puissance des signaux suivants et donner leur classe énergétique:*

	Energie	Puissance
$Rect(t).cos(t)$ 0,5×2	$E_x = \int_{-1/2}^{1/2} (1\cdot\cos(t))^2 dt = \int_{-1/2}^{1/2} \frac{1+\cos 2t}{2}$ $= \boxed{1/2 + \sin(1)}$ Energie fini	Energie fini → puissance nulle $P_x = 0$
$A.u(t)$ 0,5×2 ✓	$= \int_0^{\infty} (A)^2 dt = A^2 [t]_0^{\infty} = \infty$ Energie Infini	$P_x = \lim_{T\to\infty} \frac{1}{T} \int_0^{T/2} A^2 dt = A^2 \lim_{T\to\infty} \frac{1}{T} [t]_0^{T/2}$ $P_x = A^2/2$
$Sign(t).$ 0,5×2	$= \int_{-\infty}^{+\infty} (Sign(t))^2 = \int_{-\infty}^{+\infty} (1) dt = \infty$ Energie Infini	$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} (Sign(t))^2$ $= \lim_{T\to\infty} \frac{1}{T} [t]_{-T/2}^{T/2} = \boxed{1}$
0,5× ✓ $\delta(t-1).$ ~~0,5×2~~	$E_x = \int_{-\infty}^{+\infty} (\delta(t-1))^2 dt = \int_{-\infty}^{+\infty} \delta(t-1) dt$ $= 1$	energie fini → puissance nulle $P_x = 0.$
$u(t-1)\,e^{-at}$, $a > 0$ 0,5× ✓	$E_x = \int_1^{\infty} A^2 e^{-2at} dt = \frac{A^2}{-2a} \left[e^{-2at}\right]_1^{\infty}$ $= \boxed{\frac{A^2}{2a} e^{-2a}}$ (Energie fini	Energie fini → Puissance nulle.

Exercice 3 06/05

- On considère les deux signaux $x(t) = Rect(t/B)$ et $h(t) = A\,u(t)\,e^{-at}$ $(A > 0, a > 0)$

Représenter et calculer graphiquement le produit de convolution de $x(t)*h(t)$. Ensuite **déduire** la fonction d'inter-corrélation $C_{xh}(\tau)$.

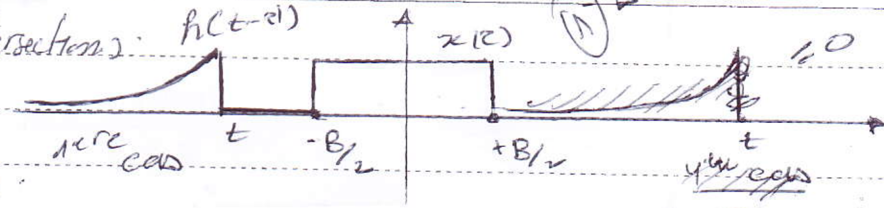
(1)

L'Expressions de le produit de convolution : $x(t)*h(t) = \int_{-\infty}^{+\infty} x(t-\tau)\,h(\tau)\,d\tau = \int_{-\infty}^{+\infty} h(t-\tau)\,x(\tau)$

(1)

1ère cas si $t < -B/2$ (pas d'intersection) : $x(t)*y(t) = 0$

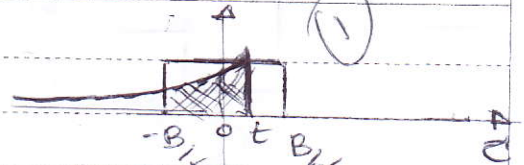
4ème cas si $B/2 < t$: (pas d'intersection)



3ème cas : $-B/2 < t < B/2$ (1)

$$x(t)*h(t) = \int_{-B/2}^{t} e^{-a(t-\tau)} \cdot 1\, d\tau = \frac{e^{-at}}{a}\left[e^{at} - e^{-aB/2}\right]$$

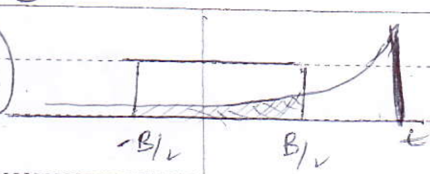
$$= \frac{1}{a}\left[1 - e^{-a(t+B/2)}\right]$$



2ème cas $B/2 < t$ (1)

$$x(t)*y(t) = \int_{-B/2}^{B/2} e^{-a(t-\tau)} \cdot 1\, d\tau = \frac{e^{-at}}{a}\left[e^{a\tau}\right]_{-B/2}^{B/2}$$

$$= \frac{e^{-at}}{a}\left[e^{aB/2} - e^{-aB/2}\right]$$



- La fct d'intercorrélation = fct de corrélation car $x(t)$ fct paire

Exercice4

Donnez l'expression de la transformation de Fourier d'un signal non périodique $x(t)$, $X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt$ (1)

- On considère les deux signaux, $x(t) = tri(t)$ et $y(t) = \sin(2\pi t)$

Calculer la transformer de Fourier de $x(t)$ (1,5)

$x(t) = tri(t) = rect(t) * rect(t)$

$TF\{x(t)\} = X(f) = TF\{tri(t)\} = TF\{rect(t)*rect(t)\} = TF\{rect(t)\} \cdot TF\{rect(t)\}$

$= sinc(f) \cdot sinc(f) = sinc^2(f)$

Calculer la transformer de Fourier de $y(t)$ (1,5)

$y(t) = \sin(2\pi t) = \frac{1}{2j}\left(e^{j2\pi t} - e^{-j2\pi t}\right) \Rightarrow TF\{y(t)\} = TF\{\frac{1}{2j} e^{j2\pi t}\} - TF\{\frac{1}{2j} \cdot e^{-j2\pi t}\}$

$= \frac{1}{2j}\left[TF\{e^{j2\pi t}\} - TF\{e^{-j2\pi t}\}\right] = \frac{1}{2j}\left[\delta(f+1) - \delta(f-1)\right]$

Calculer $X(f)*Y(f)$ (1,5)

$X(f) * Y(f) = TF\{x(t) \cdot y(t)\} = sinc^2(f) * \frac{1}{2j}\left[\delta(f+1) - \delta(f-1)\right]$

$= \frac{1}{2j}\left[sinc^2(f+1) - sinc^2(f-1)\right]$

$x(t)*\delta(t-t_0) = x(t-t_0)$

$\sin(t) = \frac{1}{2j}\left(e^{jt} - e^{-jt}\right)$, $TF(e^{-j2\pi f_0 t}) \rightarrow \delta(f-f_0)$, $rect(t)*rect(t) = tri(t)$, $TF(rect(t)) \rightarrow sinc(f)$